Supporting Mathematical Proficiency

Strategies for New Special Education Teachers

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Special educators can play an essential role in the development of students' understanding of and capacity to do mathematics. Whether a special education teacher's goal is to help kindergartners develop counting skills or to support 10th graders in constructing geometric proofs and whether instruction will occur in coteaching, consulting, or pullout settings, teaching mathematics is complex. It is not sufficient for teachers to understand how to do the mathematics themselves; teachers must also integrate multiple bodies of knowledge to facilitate equitable mathematics learning experiences for all students, especially those with disabilities. Fortunately, teachers do not have to resort to trial and error to navigate the complexity of mathematics teaching on their own.

What It Means to Do Mathematics

The purpose of this article is to highlight high-leverage mathematics teaching practices that can be immediately implemented by new special education teachers. Yet, before we make an argument for why a specific mathematics teaching practice is effective, we first establish a vision for what it means to *do mathematics*. In other words, what does it mean to learn mathematics? The term *mathematics* has been defined as "the science of pattern and order," and the doing of mathematics has been described by active verbs such as *exploring, explaining, constructing, developing*, and *predicting* (Van de Walle, Karp, & Bay Williams, 2016). Mathematics learning involves active engagement in logical reasoning and sense making and is not defined by the facts that a student knows or even what procedures a student can memorize. To illustrate this distinction, consider the difference between a student reciting the Pythagorean Theorem ($a^2 + b^2 = c^2$) and the distance formula $\sqrt{(x_2 - x_1) + (y_2 - y_1)^2}$ and a student demonstrating how the Pythagorean theorem relates to the distance formula and how either one can be used to find the distance between two points on a coordinate plane. The first task of reciting the formulas is about memorization and does not provide an opportunity for the student to engage in the doing of mathematics, but the second task demands sense making and gives the student the opportunity to do the work of a mathematician. The student engages in the science of patterns and order by identifying the underlying structure between the two formulas and explaining how they are related.

What It Means to Be Proficient in Mathematics

Having a vision for what it means to engage in the work of mathematics is only part of the story. Teachers also need to understand what it means for students to be proficient in mathematics. The National Research Council (2001) used the best available research on mathematics learning to identify five strands of mathematical proficiency: adaptive reasoning, strategic competence, conceptual understanding, productive disposition, and procedural fluency. The fact that there are five strands alerts teachers to the reality that success in mathematics is multifaceted. This is part of what makes mathematics teaching so complex. From low scores on formal assessments, teachers may conclude that some students have low procedural fluency in a particular content strand in mathematics. However, developing procedural fluency alone is not likely to result in sufficient progress for these students. According to the National Research Council, it is important to facilitate opportunities for struggling students to (a) strengthen their dispositions toward approaching mathematical tasks, (b) develop conceptual understanding of the target topics, (c) increase their ability to be strategic in problem solving, and (d) improve their adaptive reasoning skills. Only when all strands are incorporated will students have optimal opportunities to learn the target mathematics content.

Complementing the five strands of proficiency (National Research Council, 2001), the National Council of Teachers of Mathematics (2000) outlined five process standards that describe the ways in which students should go about learning mathematics content. Then, in 2010, the National Governors Association Center for Best Practices and the Council of Chief State School Officers (NGA & CSSO) drew on the five strands of proficiency and the five process standards to create the eight mathematical practices delineated in the Common Core State Standards for Mathematics. These eight standards describe the mathematical habits of mind that facilitate optimal learning in mathematics. These practices summarize how students should go about learning the "what" of mathematics. Figure 1 provides a side-by-side comparison of the strands of proficiency, the process standards, and the mathematical practices. In its entirety, this figure supports a robust vision of what it means for students to do mathematics.

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Unfortunately, the vision established in Figure 1 of what it means for students to learn and succeed in mathematics has not always been thought to be applicable to students with disabilities (SWD). Although each student has a unique cognitive, social, and emotional profile, many SWD who struggle with mathematics demonstrate

Figure 1. Robust vision for how students should engage in mathematics

Strands of mathematical proficiency[a]	Process standards[b]
• Adaptive reasoning • Strategic competence • Conceptual understanding • Productive disposition • Procedural fluency.	• Problem solving • Reasoning and proof • Communication • Connections • Representations

Mathematical practices[c]

1. Make sense of problems and persevere in solving
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

[a]See National Research Council (2001). [b]See National Council of Teachers of Mathematics (2000). [c]See National Governors Association Center for Best Practices & Council of Chief State School Officers (2010).

one or more of the following common characteristics that can be barriers to full participation in this vision of mathematics learning: learned helplessness, passive learning, knowledge and skill gaps, math anxiety, mathematics learning disabilities, memory deficits, attention deficits including hyperactivity and impulsivity, metacognitive thinking disabilities, processing disabilities, and reading disabilities (Allsopp, Lovin, & van Ingen, in press). Although these barriers may initially hinder students from participation in the mathematical processes and practices, they are not insurmountable. When the barriers are adequately addressed, SWD are capable of developing proficiency and engaging in the practices and processes that are central to mathematics (Gersten et al., 2008).

It is essential for new special education teachers to be committed to a robust vision of what it means for all students—especially SWD—to engage in mathematics. With such a vision in place, special education teachers can engage in actions and practices that support overcoming these barriers so that SWD can fully participate in learning mathematics. Most certainly, there are specific and different forms of pedagogical knowledge and pedagogical content knowledge that are necessary to teach mathematics effectively to a student with a processing disorder as opposed to a student with an intellectual disability or an emotional/behavioral disorder. In this article, rather than focusing on specific practices matched to specific profiles, we recommend two high-leverage practices that have been shown to be effective for students with a range of disabilities: teaching with explicitness and teaching mathematics metacognition.

Two mathematics teaching practices with strong support from the special education and mathematics education communities are teaching with explicitness and teaching mathematics metacognition.

Teaching With Explicitness

Identifying the focus of instruction is essential to effective mathematics instruction. Teachers develop this focus by considering the needs of the student and clearly identifying appropriate learning objectives. Once clear objectives are identified, explicit mathematics instruction consists of teaching practices that help to highlight and make clear to students the significant mathematical ideas embedded in these learning objectives (Allsopp et al., in press). Two explicit teaching techniques that will help teachers make mathematics accessible to their students include employing task analysis and using visuals.

Task Analysis

Task analysis is a research-supported effective practice for SWD (Browder, Trela, & Jimenez, 2007; Cooper et al., 2007; Szidon & Franzone, 2009). In short, a task analysis identifies the actions necessary to understand a target concept or to carry out a given skill. It breaks an academic behavior, skill, or concept into smaller steps, teaches each step, and connects one step to another (*chaining*) until students are able to engage in and understand the whole academic concept or skill. As adults, we may not be conscious of the specific actions in which we engage and problem-solve "automatically." This is akin to reading: We see a word and then read it without thinking about the various features of the word that allow us to read it and make meaning of it (phonology, phonics, semantics, etc.). Szidon and Franzone (2009) outlined general steps for implementing a task analysis that can help teachers unpack what has become automatic for them:

1. Identify the target behavior, skill, or concept.
2. Determine the necessary prerequisite skills and knowledge.
3. Break down the behavior/skill/concept into parts.
4. Review to ensure the behavior/skill/concept is completely analyzed and all important aspects (behavioral and/or cognitive) are included.
5. Decide how best to teach the task analyzed behavior/skill/concept based on the student's needs.
6. Implement instruction and monitor success.

Figure 2. Sample task analysis for multidigit subtraction

Task analysis steps	453 – 275 = ?
1. Student estimates the difference.	453 – 275 is about 450 – 250 which is 200, so the answer should be about 200
2. Student demonstrates notion of expanded notation for each multidigit number (place value).	400 + 50 + 3 - (200 + 70 + 5)
3. Student demonstrates and can accurately regroup when necessary.	300 + 140 + 13 - (200 + 70 + 5)
4. Student accurately subtracts numbers in corresponding place values.	100 + 70 + 8 = 178
5. Student adds the amount.	275 + 178 453
6. Student confirms final answer and compares to estimation.	453 – 275 = 178

Figure 2 provides an example of a task analysis for multidigit subtraction with place values. Note that there are multiple strategies to do multidigit subtraction, such as equal additions and adding up from the lower number. In this example, we focus on one strategy: emphasizing the place value of the digits.

Mathematical task analysis can aid instruction in several ways. First, it heightens awareness of the key features that students need to understand to engage with the larger mathematical task. Second, it is helpful because SWD often benefit from instruction that focuses on smaller chunks of content and that explicitly engages students in connecting the chunks to make sense of the whole concept or skill. Finally, a task analysis provides teachers with a process for thinking about how they can differentiate their instruction according to a student's readiness. Table 1 shows an example of how a teacher might record where three students (Cassandra, Michael, and Tommy) are with respect to the task analysis shown in Figure 3 (based on results of an informal assessment) and identify specific instructional accommodations and modifications for them.

Note that students can become so focused on mastering the substeps that they fail to consider why they are using the steps or how each step contributes to a larger skill or concept. As a result, they miss the big picture and do not develop the kind of understanding and proficiency described earlier. This potential pitfall can be avoided when teachers build in time—before, during, and after a task analysis—to remind students of the overall skill or concept and how each step fits into the larger process. In particular, consistently using estimation as a first step in any computation-related task is a great way to engage students' sense making and help students look at the big picture before diving into each substep. Referring back to this estimation at various places in the process can remind students of the overall skill or concept. In addition, a flowchart of the task analysis (Figure 3) can be posted, on which the teacher can highlight and add checkmarks as students complete each part. Coupling this chart with questions that focus on the reasoning behind the steps and how each step contributes to the overall skill or concept is paramount. In this way, teachers can use the chart to help students connect what they are working on now to what they have already learned and to what they will learn next, providing them with a way to self-monitor their progress. By utilizing a mathematical task analysis as a continuous component of teaching mathematics, special educators will be much better situated to anticipate and respond to their students' gaps in knowledge and skills.

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Use Visuals

Visuals support students to conceptualize or "visualize" the key features of a mathematics concept or skill and are another effective approach for explicitly teaching mathematics for SWD (Gersten et al., 2008). Different

Table 1. Student Needs Related to Multidigit Subtraction Strategy With Place Values

Knowledge/skill needed	Knowledge/skill gaps for students	Possible accommodations/ modifications
1. Expanded notation	Cassandra often fails to strategically identify the place value of digits prior to subtracting.	Teach students to use or draw place value mat and notation; provide cue sheet with model during practice.
2. Regrouping/place value	Michael frequently subtracts the smaller-value digit from the larger-value digit regardless of whether the smaller digit is in the subtrahend.	Use a story problem to put the numbers and operation into a context so that the student can act out the situation. Students may initially find it easier to model a takeaway subtraction situation but should be given opportunities to work on other types of subtraction situations (e.g., comparisons). With meaningful contexts and concrete materials, help students develop discrimination skills to determine when regrouping is required and when it is not. Continue to provide opportunities for them to explain how they know when they need to regroup and when they do not.
3. Subtracts numbers in corresponding place values	Tommy subtracts accurately most of the time but sometimes has difficulty aligning the numbers correctly; Cassandra and Michael are inconsistent.	Provide graph paper for number alignment; provide calculator option.
4. Checks answer (adds the resulting answer and the amount being subtracted)	All three students need prompting at times (i.e., when having difficulty identifying which numbers to add).	Provide self-monitoring cue sheet with each step of the process and highlight this step. Students check off each step as they complete it.
5. Checks reasonableness of answer using estimate	Tommy often proceeds to the next equation before checking the reasonableness of his solution.	Same as above; monitor/prompt Tommy and reinforce his checking.

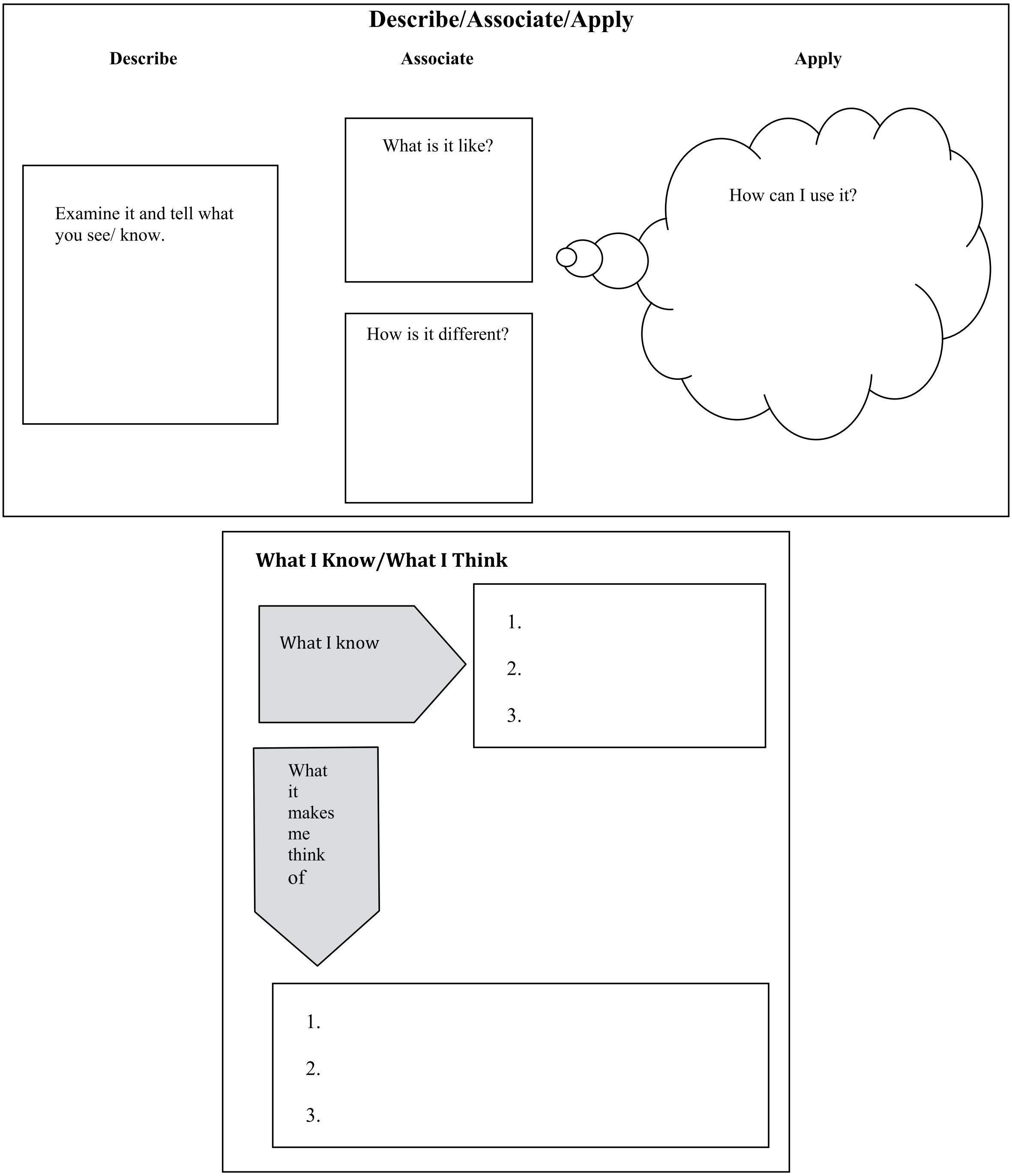
types of visuals— such as graphic organizers, diagrams, drawings, pictures, concrete materials, and cuing devices—can be utilized to make mathematical ideas and skills cognitively accessible for students.

Graphic organizers help students to organize their thinking and make connections among mathematics concepts or skills. Figure 3 shows examples of graphic organizers that prompt students to describe and apply target concepts or skills and associate them with other mathematical ideas.

Visual cuing helps students attend to and discriminate key features of a mathematics concept or skill. Figure 4 shows examples of different cuing strategies, which include color-coding, written words to describe meaning (e.g., initial meaning of fractions), cue sheets (e.g., DRAW strategy), and pictures (e.g., long division). To create effective visuals such as these, the teacher must identify and highlight key features of the mathematics concept or skill that make it distinct and then provide students with visual anchors to attend to, understand, and remember the concept or skill.

Figure 5 shows a visual that uses a concrete-to-representational-to-abstract (C-R-A) sequence of instruction. C-R-A instruction can benefit SWD by helping them develop conceptual understandings of mathematics procedures by connecting representations that range from the concrete to the abstract. Another form of visuals, schematic diagrams, is well documented as an effective instructional practice to use with struggling learners (Hegarty & Kozhevnikov, 1999; Krawec, 2012; Powell, 2011; van Garderen & Montague, 2003). In particular, *schematic representations* (diagrams that represent quantities through proportional or spatial relationships) as opposed to *pictorial representations* (drawings of objects that represent their visual appearance) are especially valuable in helping

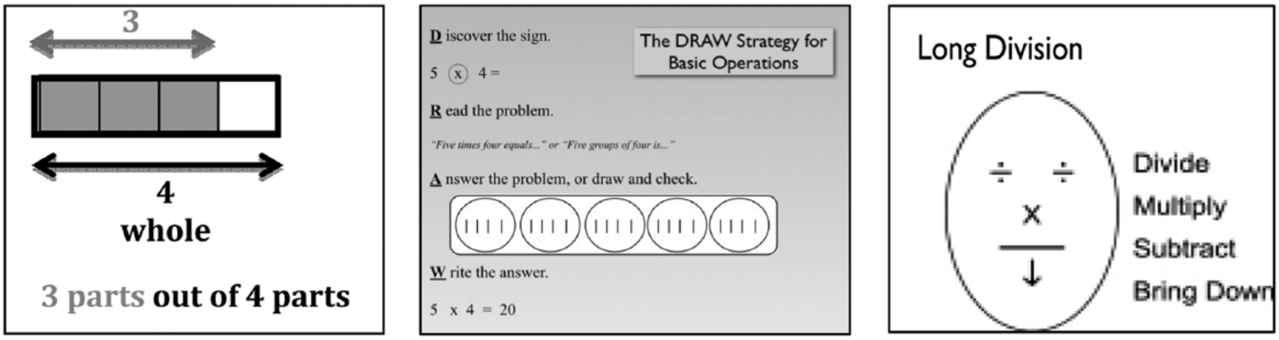
Figure 3. Graphic organizers to support students making explicit connections among related mathematical ideas



students consider quantities and the relationships among them. Consequently, schematic representations are effective at helping students make sense of situations (e.g., Brenner et al., 1997; Stylianou & Silver, 2004). To illustrate the difference between the two types of representations, Figure 6 provides examples of pictorial representations and schematic representations for two story problems.

Research has provided strong evidence that when struggling learners are expected to use visuals to reason about and solve problems, as opposed to using visuals to just show the answer after solving a problem in another way, they are more successful in making sense of and solving mathematical problems (Gersten et al., 2008). However, students must be explicitly taught how to create and use visual representations with schematic characteristics. Otherwise, struggling learners tend to use pictorial representations that lack strong relational and numeric components (Hegarty & Kozhevnikov, 1999; Krawec, 2012; van Garderen &

Figure 4. Sample visuals that cue students to key features of the mathematics idea



Note. For DRAW strategy, see Mercer & Miller (1991).

Montague, 2003). Using think-alouds, teachers can explicitly model how they identify and represent the quantities in a problem so that the quantitative and spatial relationships are evident in the representation. Teachers can use specific techniques to help students attend to important aspects of a mathematical representation or process or focus on important ideas such as the relationships among quantities:

- Color-coding—using color to assist students in visualizing relationships between numerical representations and diagrams
- Labeling—including written words next to numerical representations that explain the meaning of the numerical representation
- Directional arrows—providing students with visual-spatial support by showing them where to begin and where to go to next
- Highlighting—emphasizing particular numbers, words, drawings

Teaching Mathematics Metacognition

Teaching SWD how to "think about their thinking with mathematics"—that is, to become metacognitively aware of their reasoning—is important to their overall mathematical success. To help focus students' attention on their mathematical reasoning, teachers can use purposeful strategy instruction and promote discourse about mathematics.

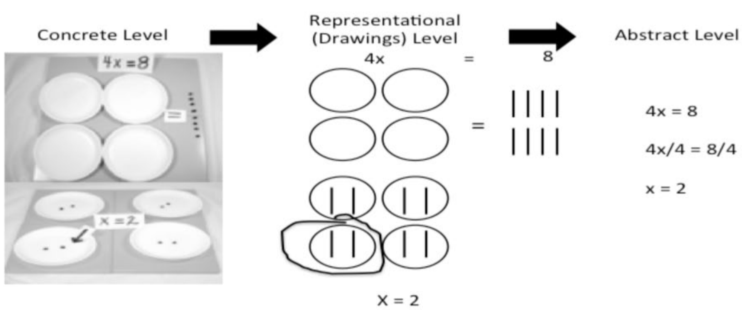
Mathematics Strategy Instruction

The goal of strategy instruction is to teach students how to be strategic when approaching mathematics tasks and to discriminate which strategies are best for particular tasks. Two approaches to strategy instruction are (a) teaching general problem-solving strategies and (b) teaching concept- and skill-specific strategies.

Teaching general problem-solving strategies. Metacognition allows students to monitor and regulate learning (Flavell, 1979). Research has shown that students develop two types of metacognition: first, *declarative metacognition*, as they gain knowledge about strategies and their own strengths and weaknesses; second, *procedural metacognition*, as they learn to monitor their performance related to those strategies (Schraw, 1994).

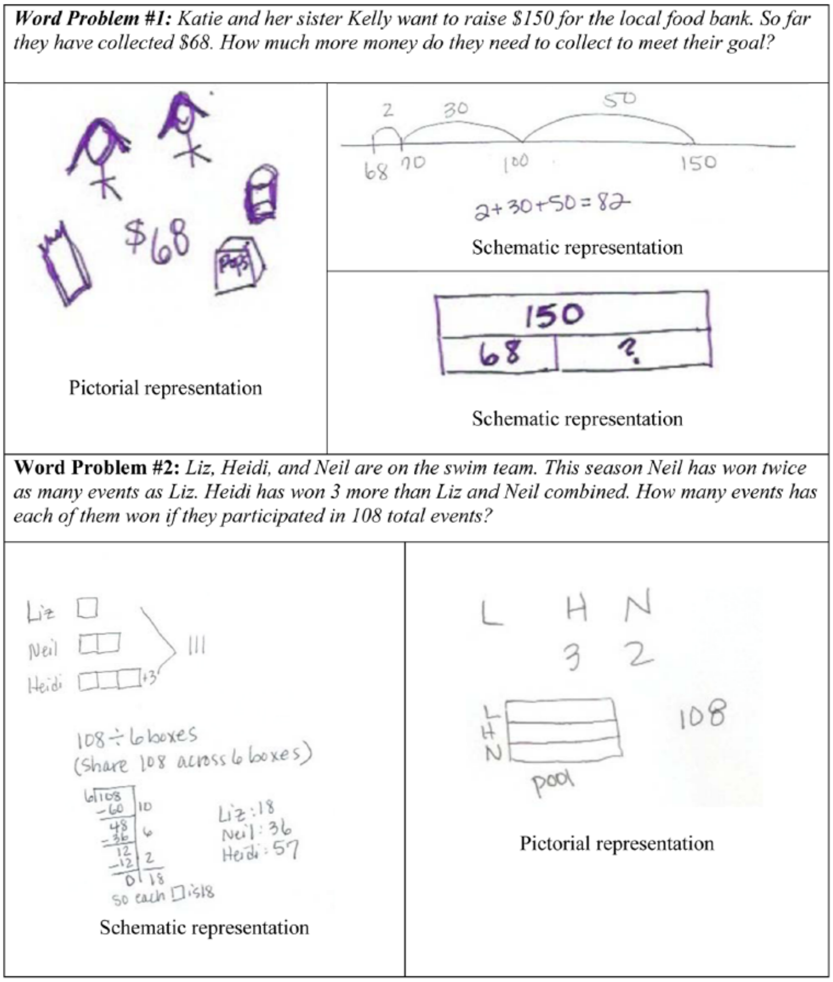
In the field of mathematics education, metacognition has often been associated with the problem-solving process (e.g., Kramarski, 2008; Schneider & Artelt, 2010; Silver, 1982; Verschaffel et al., 1999). In fact, studies have shown that the development of problem-solving metacognitive skills is linked to substantial increases in mathematics performance (Schneider & Artelt, 2010). This means that students are likely to benefit significantly when teachers take time to support students' metacognition related to problem solving.

Figure 5. Concrete materials and drawings as visuals in a concrete-to-representational-to-abstract sequence of instruction



Note. Solutions that use schematic representations demonstrate an understanding of the quantities involved and their relationship. Pictorial representations tend not to be useful for finding solutions.

Figure 6. Students' strategies for two word problems



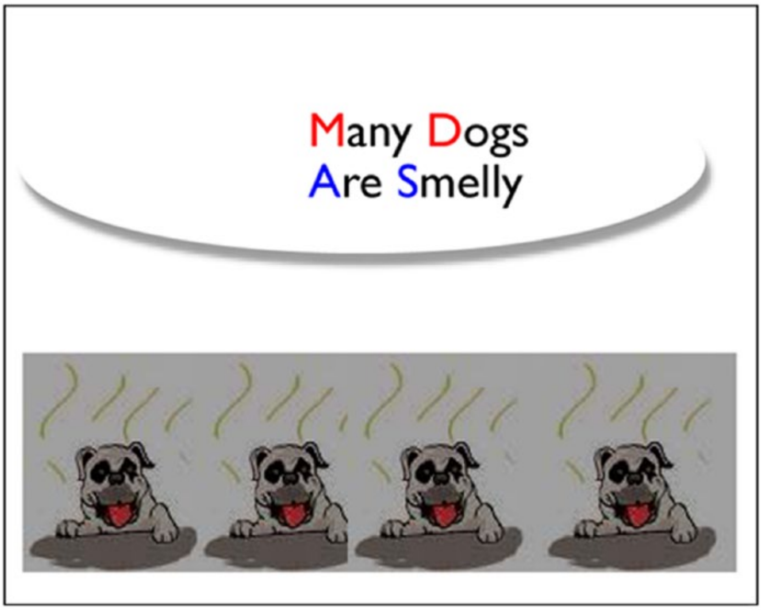
Teachers can help students develop declarative problem-solving metacognition by directly and explicitly teaching students about the problem-solving process. Numerous frameworks have been developed to help teachers do just this, and most are variations of Polya's (1945) seminal work on the four principles of problem solving: understand the problem, devise a plan, carry out the plan, and look back. The purpose of such a four-step framework is to break the larger problem-solving process down into the essential substeps that are integral to the success of the entire process. Whereas students are often initially overwhelmed by the overall problem-solving process, they can begin to make progress by working through the process one step at a time. Yet, it is not enough for students to know the names of the four steps; they also need to know what it looks like to engage in each step. Students need a repertoire of strategies that they can leverage to "devise a plan" to solve the problems. Useful strategies to introduce to students include the following: acting the problem out, drawing a diagram (as in schematic diagrams), making a table, creating a graph, and writing an equation.

Once students gain declarative knowledge of the steps, teachers can help them develop the procedural metacognition to monitor their progress through the process. Teachers can model procedural metacognition by engaging in think-alouds in which they talk through monitoring of their own problem solving (Mayer, 1987; Pressley, 1990); then, they can have students engage in think-alouds with one another. Teachers can also create tools, such as a flowchart or a checklist, that support this type of monitoring.

Teaching concept- and skill-specific learning strategies. In addition to learning general problem-solving strategies, SWD often benefit from learning strategies that are specific to a particular mathematics concept or skill. These learning strategies come in different forms, such as mnemonics, acronyms, pictures, and visual imagery. Such learning strategies provide students with ways to better understand and apply specific mathematics concepts or skills. In turn, this supports SWD to develop the requisite knowledge and skills to effectively apply general problem-solving strategies. To be effective, mathematics learning strategies should include the following: (a) a limited number of parts or steps (e.g., between three and seven), (b) an accurate portrayal of the mathematics concept or skill, (c) cues to help students attend to and understand each part or step of the strategy, and (d) features that make it memorable. The use of visuals—part of the first effective mathematics practice that we discuss in this article—can be integrated with the teaching of mathematics metacognitive strategies to support students' learning by helping them remember concept- or skill-specific learning strategies.

Figures 7 and 8 provide additional examples of mathematics learning strategies that relate to specific mathematical ideas or skills. Whereas "long division" strategy in Figure 4 is a visual with cuing that provides students support to remember the steps for completing a computation using the standard long division procedure, Figure 7's mnemonic assists students in recalling the order of operations (multiply, divide, add, subtract), and the acrostic FIND in Figure 8 supports students in identifying the place value of digits in a multidigit number and when multidigit numbers are added, subtracted, multiplied, and divided. In addition, cue sheets can help students utilize the strategy during independent practice and can be faded as students become proficient with the strategy.

Figure 7. A mnemonic learning strategy with visual



Note. In the acronym the M and D are color coded the same color meaning either operation can be done first and specifically should be done as they appear left to right in the given expression. Likewise, A and S are the same color so either operation can be done first, that is, after multiplication and division.

Engage Students in Discourse and Elaboration

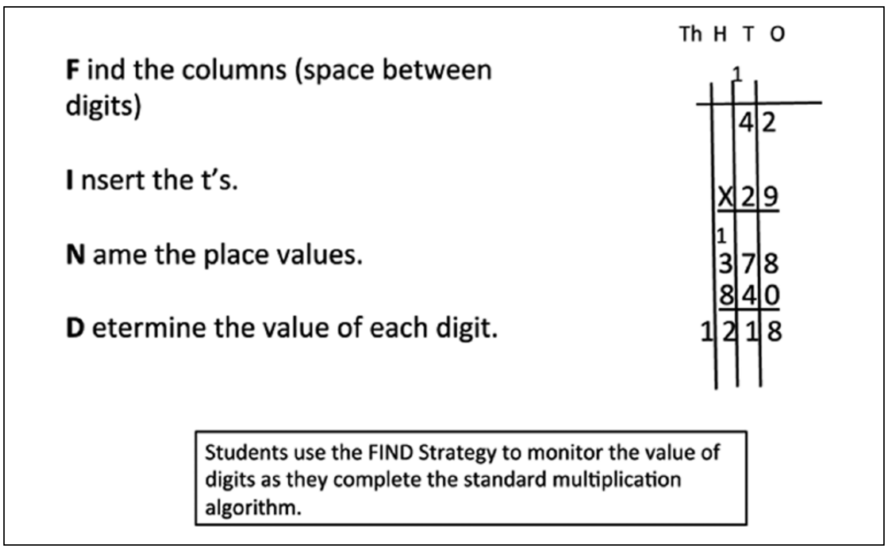
SWD can enhance their mathematical understandings when they are provided opportunities to engage in discourse and elaboration about their mathematical reasoning. We have described doing mathematics as an active engagement in logical reasoning and sense making. This involves making conjectures, sharing justifications for ideas, and explaining to others why something makes sense. Because mathematics is about reasoning and not memorizing, students need to be engaged in these activities to help them refine and improve their understanding. The related Mathematical Practice 3 of the Common Core State Standards ("Construct viable arguments and critique the reasoning of others") states that mathematically proficient students "justify their conclusions, communicate them to others, and respond to the arguments of others. . . . Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments" (NGA & CSSO, 2010, pp. 6–7). When provided with the appropriate supports, SWD can productively engage in these activities.

This type of communication does not come automatically, so teachers need to teach classroom norms in an explicit manner so that students know what is expected of them. These norms should seek to make students feel comfortable sharing their ideas and critiquing others' ideas. Here are some tips for doing so:

- Require students to explain and justify their answers as well as how they got them (whether the answers are correct or not).
- Highlight how explaining their answers and strategies contributes to everyone's learning.
- Create a culture where mistakes are seen as valuable because they help the community avoid potential errors/misconceptions and improve understanding.
- Communicate the expectation that students are to listen to and attempt to understand others' explanations and ask clarifying questions to better understand the explanations.
- Expect students to offer suggestions to improve others' explanations.
- Introduce students to the idea of critiquing others' ideas by strategically sharing an anonymous student's work that includes a misconception that is important for students to unpack.

SWD will need support to engage in classroom discourse. Consider how to

Figure 8. An acrostic learning strategy for place value



Note. Students use the FIND strategy (Mercer & Miller, 1991) to monitor the value of digits as they complete the standard multiplication algorithm.

place visual aids, especially relevant mathematical vocabulary, so that students can easily reference these for their own explanations and when offering suggestions to others.

To promote students' talking about mathematics, teachers can use the five "talk moves" proposed in *Classroom Discussions: Using Math Talk to Help Students Learn* (Chapin, O'Connor, & Anderson, 2009). These talk moves are meant to support and advance students' mathematical thinking and learning:

1. Revoicing what a student has said to ensure that it was heard or interpreted correctly (e.g., "So you are saying that you added 10 to each number?")
2. Asking a student to restate another student's reasoning—this strategy helps students pay attention to one another's reasoning
3. Applying one student's reasoning to another's (e.g., "Do you agree or disagree and why?" "What do you think about Jonah's explanation?")—this encourages students to pay attention to others' contributions
4. Prompting further comments, questions, or extensions (e.g., "Can someone expand on Teri's idea?")
5. Waiting—although it may at first be uncomfortable because of the silence, students should be given time to process the teacher's question and formulate a response; teachers can tell students that they will have some "think time" before anyone is asked to offer ideas

When in a whole class setting, teachers can give time for students to share their ideas with a partner or in a small group. This allows struggling learners time to process their thoughts before sharing with the whole class. It is likely that teachers will need to be intentional about teaching students how to engage in discussions with their partners, small groups, and whole class. For example, teachers could say, "In your partner talk, I expect to hear you using the words *equation*, *slope*, and *x*- and *y-intercept*." Teachers can also provide guidance in what students should be doing when listening to someone else's ideas. For example, a teacher might say, "When your partner is sharing her ideas, you need to be looking at her while she talks and thinking about how her ideas are similar to and different from your ideas." As students engage in partner or small-group talk, teachers can go around and listen to their discussions. As they monitor discussions, teachers can point out what students are doing well, how they can improve their explanations, and what questions they ask of one another. Students with language expression difficulties can demonstrate their understandings through communication that does not rely only on speaking. They can support their communications with concrete materials, schematic diagrams, video, or other means.

Three Reliable Resources

It is important that teachers have reliable resources where they can go to learn more about effective mathematics practices. Table 2 describes three such reliable resources that teachers can access for their continued learning and professional development.

Final Thought

With this article, we hope that special education teachers will not be satisfied with merely considering the mathematics skills profile of SWD and that teachers will not measure students' success alone by their ability to compute long division or solve a quadratic equation. Rather, we hope that special education teachers will assess and support the engagement of SWD in mathematical practices and processes. Student growth in the practices may be slow and less visible than growth in computation. Nevertheless, it will be the practices, like roots beneath a tree, that create a solid foundation upon which students will

Table 2. Resources Related to Mathematics Instruction and Students With Disabilities

Resource	Description
Center on Instruction, http://centeroninstruction.org/	The Center on Instruction provides educators with a clearinghouse for research related to evidence-based practices, including practitioner guides, professional development modules, and other tools to help teachers implement effective practices. Mathematics-related instructional practices for students with disabilities can be located by clicking on the "Special Education" tab on the left menu and selecting "Math Instruction and Intervention." Additional resources can be found by clicking on the "Science Technology Engineering Math" tab and then selecting "Mathematics."
Institute of Education Sciences (2009)	*Assisting Students Struggling With Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools* provides teachers with eight recommendations for intervening early to support struggling with mathematics. It includes both text and video support.
National Council of Teachers of Mathematics (2014)	*Principles to Actions* is "a significant step in articulating a unified vision of what is needed to realize the potential of educating all students—under any standards or in any educational setting. Most important, it describes the actions required to ensure that all students learn to become mathematical thinkers and are prepared for any academic career or professional path that they choose" (p. vii).

realize their full potential as mathematics students. We have highlighted two high-leverage research-supported mathematics teaching practices and associated teaching techniques that can benefit students. Teachers can start with these practices, but we hope that they do not stop there. We encourage special educators to use the principles from this article to launch a career of constant inquiry, discussion, and learning with colleagues, related to how to meet the mathematics learning needs of SWD.

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